(1) The large octopus (probably *Octopus dofleini*) that lives under the FHL dock is extremely sensitive to blood in the water; it will sense and respond to a single red blood cell. During an all-apprentice swim off the docks, one exuberant swimmer unknowingly knicks a foot on a barnacle (probably *Balanus glandula*), releasing a million red blood cells at 3 meters distance from the octopus. Assume that:

- Particles only move by diffusion (i.e. convective flow is negligible).
- The radius of a spherical red blood cell is about 4 microns.
- The water temperature is 9ºC.
- The viscosity of sea water at 9ºC is 1.4×10^{-3} Pa-s.
- We can model this as diffusion in 3D with boundaries infinitely far away (i.e. neglect the surface of the water, the ocean floor, etc)
- The octopus is very large, subtending an angle of π/4 steradians (i.e. 1/16th of the sphere at 3 meters.
- Octopus attack dynamics are infinitely fast.

In class we discussed the mean-square displacement and probability distribution for a 1D random walk. In 3D, with radial coordinate *r*, these are

$$< r^2 >= 6Dt \quad ; \quad p(r,t)dr = \frac{1}{\sqrt{12\pi Dt}} e^{-r^2/12Dt} dr$$

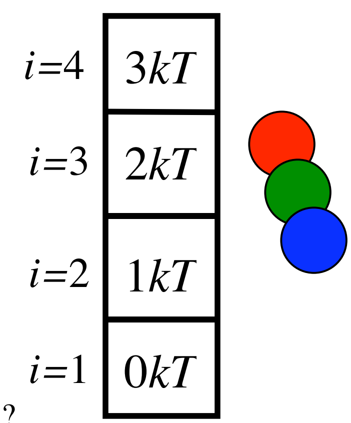
and we can solve for *t*, the mean first passage time to distance *r* as

$$t = \frac{r^2}{6D} \qquad \text{; “first passage time” or “mean time to capture”.}$$

The assignment (at last, you say):

- Calculate the first passage time to 3 meters distance for a single red blood cell. Should we wait?
- What is the probability of finding this single blood cell further than 3 meters away after the first passage time?
- What is the probability (consider all particles of blood and the solid angle of the octopus) of an attack at time *t*=45 minutes, the longest anyone has ever stayed in the water? Do you feel better now?

(2) Revisit our counting of states with discrete boxes. Now, however, there are 4 boxes and each box has the indicated energy. Count the number of possible states and distributions subject to the constraints



$$E_{Total} = \sum_{i=1}^{4} E_i = 3kT \quad ; \quad N_{Total} = \sum_{i=1}^{4} N_i = 3$$

i.e. the sum over all boxes of the energy in each box must equal the total energy, and the sum over all boxes of the number of particles in each box must equal the total number of particles.

How much more likely is distribution $s = \{1,1,1,0\}$ than $s = \{2,0,0,1\}$?

(3) No one seemed much concerned in class that there is a non-zero probability of being run-through by an air molecule of extraordinary energy, except me. Even an energy 120 times the average scares me. Soothe my fears (maybe) by calculating the probability that a particle in our lecture space will be moving with that much energy in any coordinate direction. Note:

- Our lecture space is about 5 x 5 x 4 m^3.
- An "air particle" has mass of 29 Daltons (atomic mass units).
- 1 Dalton = 1.66×10^{-27} kg.
- I calculate ~42 moles per cubic meter using the ideal gas law.
- Use $p(v) = \frac{1}{\sqrt{2\pi kT}} e^{-mv^2/2kT}$ as the probability of finding a molecule with speed v

in some coordinate direction.

- If using Mathematica to integrate, try NIntegrate[] instead of Integrate[].